Review Derivatives, Differentiability and Continuity

Find the derivatives of the following

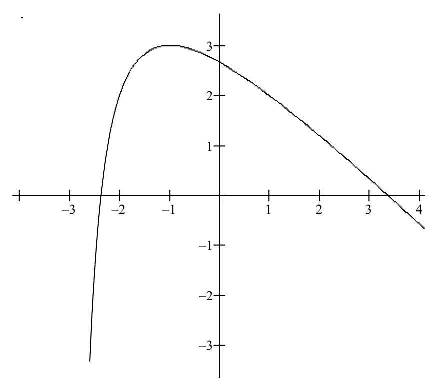
1. $f(x) = x^2 + 3x - 2$
2. $f(x) = \sqrt[3]{x} + \dfrac{1}{x}$
3. $y = \dfrac{x^2 - 4x}{\sqrt{x}}$
4. $h(x) = \dfrac{x^2}{x-1}$
5. $f(x) = x^2(\sqrt{x} + 1)$
6. $y = \dfrac{x^2}{2}$
7. $h(x) = \dfrac{x+2}{x-1}$
8. $g(x) = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$
9. y = csc(x) sec(x)
10. $f(x) = \dfrac{\sqrt{x}+2}{\sqrt{x}-2}$

11. Find the equation of the tangent line to the graph of $f(x) = \dfrac{1}{x-1}$ at the point where x = 2.

12. Find the derivative of $f(x) = 1 - x^2$ using the **definition of the derivative**.

13. Determine the instantaneous rate of change of the function $f(x) = x^3 - x$ at the point (2, 6).

Given the graph of f to the right, insert < or > between the expressions.



14. $f(0)$ ________ $f(-1)$.

15. $f'(-2)$ ________ $f'(1)$

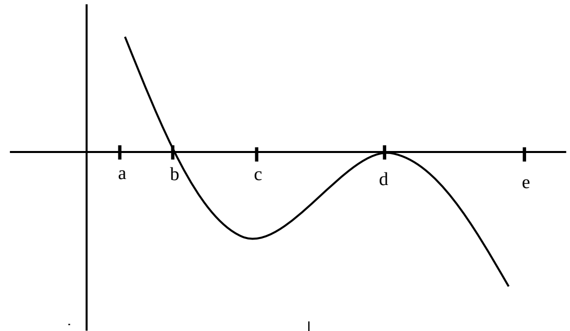
16. $f'(0)$ ________ $f(2)$

17. The volume of a balloon changes with the radius. $V = \dfrac{4}{3}\pi r^3$. At what rate is the volume changing when the radius is 4cm.

18. Find the equations of the tangent line(s) to $f(x) = x^3 + 3x^2 + 2x$ which are parallel to $y = 2x + 7$

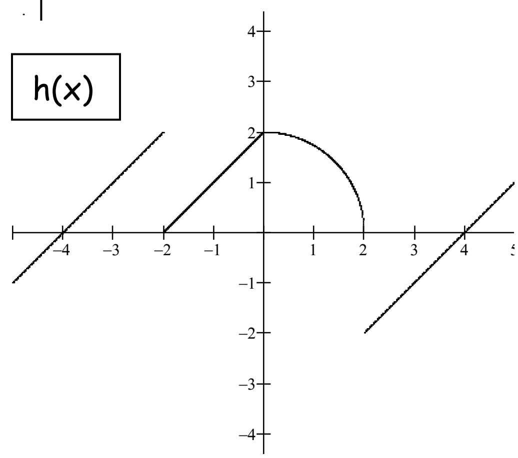
19. Mr. Bean was wondering if $h(x) = x^3 + x$ had any points where the tangents to $h(x)$ where horizontal. He took the derivative of $h(x)$, $h'(x) = 3x^2 + 1$ and set this equal to zero. When he did this he realized that the solutions to the equation were imaginary. What, if anything, can you say about $h(x)$ in this situation?

20. Suppose the curve below is the graph of $f'(x)$, the derivative of $f(x)$. It is **NOT** the graph of $f(x)$. Determine at which of the x-values and/or intervals $f(x)$ is increasing. Explain your answer.



21. Determine the x-values at which $h(x)$ is not continuous.

22. Determine the x-values at which $h(x)$ is not differentiable.



23. Given:

g(3)	g'(3)	h(3)	h'(3)
4	-2	3	π

Find f'(3) for the following:

a. f(x)= $4g(x) - \frac{1}{2}h(x) + 1$

b. f(x)=g(x)h(x)

c. $f(x) = \dfrac{g(x)}{2h(x)}$

Trig Differentiate

1. $f(t) = t^2 \sin t$
2. $f(\theta) = (\theta+1)\cos\theta$
3. $f(x) = \dfrac{\cos x}{x}$
4. $f(x) = \dfrac{\sin x}{x}$
5. $y = 5x \csc x$
6. $y = x\sin x + \cos x$
7. $y = x^2 \sin x + 2x\cos x$
8. $h(\theta) = 5\sec\theta + \tan\theta$

Higher level derivatives: Find f''(x) for

1. $f(x) = \dfrac{x}{x+1}$
2. $f(x) = 4\sqrt{x} - \dfrac{2}{\sqrt{x}}$

Given: f(x) is differentiable. $f(x)=\begin{cases} ax^3-6x & x\le 1 \\ bx^2+4 & x>1 \end{cases}$ *Find a and b*

Chain rule practice

a. $y=\sec(5x)-\csc(5x)$

b. $f(x)=\dfrac{\sqrt{5+x^2}}{x^4+1}$

c. $f(x)=\sin^2(5x)$

d. $h(x)=\dfrac{5}{(5x^2+3)}$

e. $y=\sin\sqrt[3]{x}$

Motion and other problems.

1. A particle moves along the x -axis so that it's position is given by $x(t)=t^3-6t^2+9t+11$

where x(t) is measured in inches and t is in seconds, $t\ge 0$. Justify all answers. Use appropriate units.

a. Find the velocity and acceleration functions.

b. When is the particle travelling left?

c. When is the particle stopped?

d. What is the average velocity from [0,3]

e. What is the acceleration at t=1?

f. What is the velocity when the acceleration is 0?

g. When is the particle speeding up? Slowing down?

2. **Yes calculator**. For [0,6] seconds, a particle moves along the x axis. The particle's position function is unknown. The velocity is given by $v(t)=2\sin(e^{\frac{t}{4}})+1$. v is ft/sec.

a. When is the particle moving left?

b. Is the speed of the particle increasing or decreasing at t=5.5?

c. Find the average acceleration from [0,6].

3. The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the *x*-axis. The velocity *v* is a differentiable function of time *t*.

Time *t* (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

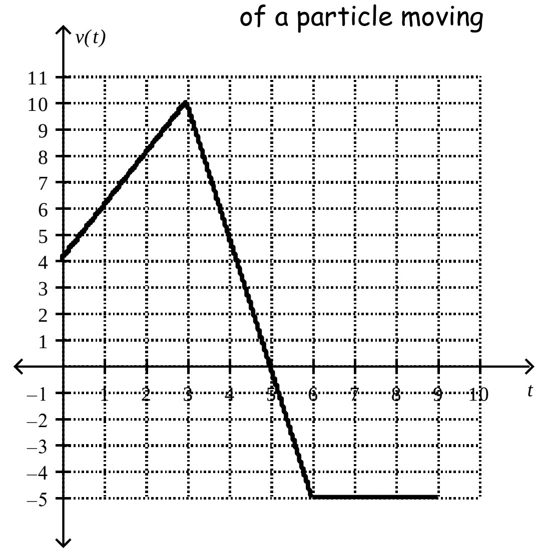
a. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

b. Is there a time during $0 \le t \le 12$ minutes when the particle is at rest? Explain.

c. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computation that lead to your answer and indicate units of measure.

d. Is the particle speeding up or slowing down at t=10. (use part c to help).

4. The graph represents the velocity $v(t)$, in feet per second, of a particle moving along the *x*-axis over the time interval $0 \le t \le 9$ seconds.



a. At $t = 4$ seconds, is the particle moving to the right or the left? Explain your answer.

b. Over what time interval is the particle moving to the left? Explain your answer.

c. At $t = 4$ seconds, is the acceleration of the particle positive or negative? Explain your answer.

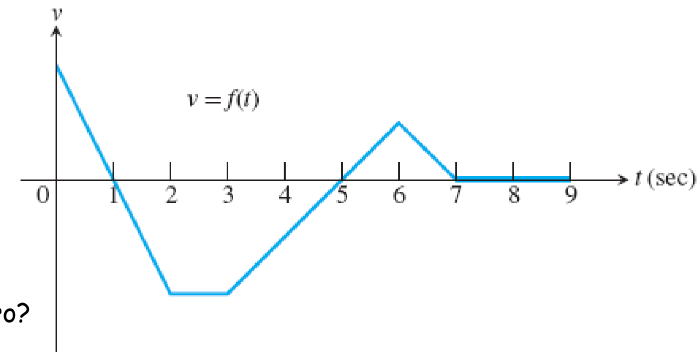
d. What is the average acceleration of the particle over the interval $2 \le t \le 4$? Show the computations that lead to your answer and indicate units or measure.

5. The accompanying figure shows the velocity $v = f(t)$ of a particle moving on a coordinate line.

When does the particle:

(a) move forward? (b) Move backward?

(c) Speed up? (d) Slow down?



When is the particle acceleration

(e) positive? (f) Negative? (g) Zero?

(h) When does the particle move at its greatest speed?

(i) When does the particle stand still for more than an instant?

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6. A particle P moves on the number line. The figure shows the position of P as a function of time(t).

a. When is P moving to the left?

b. Moving to the right?

c. Standing still?

Graph the particle's velocity and speed.